Constitutive Modeling of TRIP Steels

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Abstract

A constitutive model that describes the mechanical behavior of steels exhibiting “TRansformation Induced Plasticity” (TRIP) during martensitic transformation is presented. Multiphase TRIP steels are considered as composite materials with a ferritic matrix containing bainite and retained austenite, which gradually transforms into martensite. The effective properties and overall behavior of TRIP steels are determined by using homogenization techniques for nonlinear composites. A methodology for the numerical integration of the resulting elastoplastic constitutive equations in the context of the finite element method is developed and the constitutive model is implemented in a general-purpose finite element program. The constitutive model is used also for the calculation of “forming limit diagrams” for sheets made of TRIP steels; it is found that the TRIP phenomenon increases the strain at which local necking results from a gradual localization of the strains at an initial thickness imperfection in the sheet.

Keywords: TRIP steels; plasticity; finite element methods

1. Introduction

The TRansformation Induced Plasticity (TRIP) phenomenon has led to the development of a new generation of multiphase low-alloy steels that exhibit an enhanced combination of strength and ductility satisfying the requirements of automotive industry for good formability and high-strength. The excellent combination of mechanical properties of TRIP steels is attributed to their complex microstructure; proper heat treatment produces steels consisting of dispersed retained austenite and bainite in a ferritic matrix. The retained austenite is metastable at room temperature and, under the effect of stress and/or plastic deformation, transforms to martensite. In particular, at temperatures just above the so-called Ms temperature, transformation can be induced via stress-assisted nucleation on the same sites which trigger the spontaneous transformation on cooling, but now assisted by the thermodynamic effect of applied stress. Above a temperature Ms, the transformation stress exceeds the flow stress of the parent phase and transformation is preceded by significant plastic deformation; this is known as strain-induced nucleation and involves the production of new nucleation sites by plastic deformation. Finally, above a temperature Md, no transformation is observed prior to fracture. In the present paper we focus our attention to the temperature range above Ms and below Md, where the dominant nucleation mechanism is strain-induced.

Several authors have developed constitutive models for the mechanical behavior of dual-phase TRIP steels. Herein, we develop rate-independent constitutive equations for four-phase TRIP steels. In particular, we consider TRIP steels that consist of a ferritic matrix with dispersed bainite and austenite, which transforms gradually into martensite as the material deforms plastically. The total strain is assumed to be the sum of elastic, plastic, and “transformation” parts. The plastic part is determined by using homogenization techniques for nonlinear composites that have been developed by Ponte Castañeda ([1], [2], [3]).
2. Description of the constitutive model

We view TRIP steels as composite materials with evolving volume fractions of the individual phases. In particular, we consider four-phase TRIP steels that consist of a ferritic matrix with dispersed bainite and austenite, which transforms gradually into martensite as the material deforms plastically. The following labels are used for the individual phases: (1) for martensite, (2) for austenite, (3) for bainite, and (4) for ferrite. Let \( c^{(r)} \) \( (r = 1, 2, 3, 4) \) be the volume fraction of the phases \( \sum_{r=1}^{4} c^{(r)} = 1 \). No restriction is placed on the magnitude of the deformation and appropriate “finite strain” constitutive equations are developed. Each of the four phases is assumed to be isotropic, elastic-plastic, and is distributed statistically uniformly and isotropically. The total deformation rate \( \mathbf{D} \) is written as the sum of the elastic, plastic, and TRIP parts:

\[
\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p + \mathbf{D}^{TRIP}.
\]

The constitutive equations for \( \mathbf{D}^e \) and \( \mathbf{D}^{TRIP} \) are as in Papatriantafillou et al. [4], who considered a rate-dependent (viscoplastic) model as opposed to the rate-independent model considered here.

The constitutive equations for \( \mathbf{D}^p \) are developed as follows. Each phase is assumed to be plastically incompressible and the corresponding constitutive equations are written in terms of a “viscous potentials” of the “power-law” type \( U^{(r)} = U^{(r)}(\sigma_{eq}) \):

\[
\mathbf{D}^{(r)} = \frac{\partial U^{(r)}(\mathbf{\sigma})}{\partial \mathbf{\sigma}} = \dot{\mathbf{\varepsilon}}^{(r)} \mathbf{N} = \frac{s}{2\mu^{(r)}}, \quad r = 1, 2, 3, 4,
\]

with

\[
U^{(r)}(\mathbf{\sigma}) = \frac{\sigma_{eq}^{(r)}}{n^{(r)}+1} \left( \frac{\sigma_{eq}}{\sigma_{0}^{(r)}} \right)^{n^{(r)}+1}, \quad \mathbf{N} = \frac{3s}{2\sigma_{eq}}, \quad \sigma_{eq} = \frac{3}{2} s : \mathbf{s},
\]

\[
\dot{\mathbf{\varepsilon}}^{(r)} = \sqrt[3]{\frac{2}{3}} \mathbf{D}^{(r)} : \mathbf{D}^{(r)} = \frac{\sigma_{eq}}{3\mu^{(r)}} \left( \frac{\sigma_{eq}}{\sigma_{0}^{(r)}} \right)^{n^{(r)}}, \quad \mu^{(r)}(\sigma_{eq}) = \frac{1}{3} \mu_{0}^{(r)} \left( \frac{\sigma_{eq}}{\sigma_{0}^{(r)}} \right)^{n^{(r)}-1}, \quad r = 1, 2, 3, 4.
\]

where \( \mathbf{\sigma} \) is the stress tensor, \( \mathbf{s} \) the stress deviator, \( \sigma_{0}^{(r)} \) a reference stress, \( \dot{\mathbf{\varepsilon}}_{0}^{(r)} \) a reference strain rate, \( n^{(r)} \) the strain–rate sensitivity exponent, and \( \mu^{(r)}(\sigma_{eq}) \) is the viscous shear modulus.

There are two interesting limiting cases of the model described by equations (2), (3), and (4). The first is the linear case, in which \( n^{(r)} = 1 \):

\[
U_{L}^{(r)} = \frac{\sigma_{0}^{(r)} \dot{\mathbf{\varepsilon}}_{0}^{(r)}}{2} \left( \frac{\sigma_{eq}}{\sigma_{0}^{(r)}} \right)^{2}, \quad \mu^{(r)} = \frac{\sigma_{0}^{(r)}}{3\dot{\mathbf{\varepsilon}}_{0}^{(r)}}, \quad \dot{\mathbf{\varepsilon}}^{(r)} = \frac{\sigma_{eq}}{3\mu^{(r)}} = \frac{\sigma_{eq}}{\sigma_{0}^{(r)}} \frac{\sigma_{eq}}{\sigma_{0}^{(r)}}.
\]

The second limiting case is perfect plasticity in which \( n^{(r)} \to \infty \). Taking into account that

\[
\lim_{n^{(r)} \to \infty} A^{n+1} = \begin{cases} 0 & \text{if } A \leq 1, \\ \infty & \text{if } A > 1, \end{cases}
\]

we conclude that

\[
U^{(r)} = \lim_{n^{(r)} \to \infty} \left[ \frac{1}{n^{(r)}+1} \sigma_{0}^{(r)} \dot{\mathbf{\varepsilon}}_{0}^{(r)} \left( \frac{\sigma_{eq}}{\sigma_{0}^{(r)}} \right)^{n^{(r)}+1} \right] = \begin{cases} 0 & \text{if } \frac{\sigma_{eq}}{\sigma_{0}^{(r)}} \leq 1, \\ \infty & \text{if } \frac{\sigma_{eq}}{\sigma_{0}^{(r)}} > 1. \end{cases}
\]

Our goal is to determine the corresponding viscoplastic constitutive equation for the “composite” TRIP steel, i.e., an equation of the form \( \mathbf{D}^p = \mathbf{D}^p(\mathbf{\sigma}) \), and then consider the rate-independent limit. This is achieved in three steps.
First step
The dissipation function of the composite is defined by (Ponte [1], [2], [3])
\[
\tilde{U}(\bar{\sigma}) = \sup_{\mu^{(r)}} \left\{ \tilde{U}_L(\bar{\sigma}_{eq}, \bar{\mu}(\mu^{(r)})) - \frac{1}{2} \sum_{r=1}^{N} c^{(r)} \left[ U_L^{(r)}(\sigma_{eq}^{(r)}, \mu^{(r)}) - U^{(r)}(\sigma_{eq}^{(r)} \right) \right\}.
\]
(8)
where
\[
\tilde{U}_L = \frac{\bar{\sigma}_{eq}^2}{6 \bar{\mu}(\mu^{(r)})}
\]
(9)
is the dissipation potential of a “linear comparison composite”. The corresponding constitutive equation for the composite is
\[
\bar{D} = \frac{\partial \tilde{U}}{\partial \bar{\sigma}},
\]
(10)
where \( \bar{\sigma} \) and \( \bar{D} \) are the macroscopic stress and deformation rate tensors respectively. The effective modulus \( \bar{\mu} \) for particulate composites is defined by the well-known Hashin-Shtrikman estimate:
\[
\bar{\mu}(\mu^{(r)}) = \sum_{r=1}^{N} \frac{c^{(r)}(3\mu^{(l)} + 2\mu^{(r)})^{-1}}{c^{(r)}(3\mu^{(l)} + 2\mu^{(r)})^{-1}}.
\]
(11)
Second step
All creep exponents are set equal, i.e., \( n^{(1)} = n^{(2)} = \ldots = n^{(N)} = n \). Then the optimization problem (8) reduces to
\[
\tilde{U} = \frac{\bar{\sigma}_{eq}^{n+1}}{n+1} \sup_{y^{(r)} \geq 0} \left\{ F \left( \frac{\mu^{(r)}}{\bar{\mu}} \right) \left( \frac{3y^{(r)} + 2}{3y^{(r)} + 2} \right)^{-1} \right\}, \quad y^{(r)} = \frac{\mu^{(l)}}{\mu^{(r)}},
\]
(12)
where
\[
F \left( \frac{\mu^{(r)}}{\bar{\mu}} \right) = \frac{\sum_{r=1}^{N} c^{(r)}(3y^{(r)} + 2)^{-1}}{\sum_{r=1}^{N} c^{(r)}(3y^{(r)} + 2)^{-1}} \quad \text{and} \quad H^n \left( y^{(r)} \right) = \sum_{r=1}^{N} \left( \frac{\sigma^{(r)}_{0}}{c^{(r)}_{0}} \right)^n \left( y^{(r)} \right)^{n+1}.
\]
Third step
We consider the limit \( n \to \infty \) and (12) takes the form
\[
\tilde{U} = \begin{cases} 0 & \text{when } \frac{\bar{\sigma}_{eq}^2}{\bar{\mu}} \sup_{y^{(r)} \geq 0} \left\{ F \left( \frac{\mu^{(r)}}{\bar{\mu}} \right) \left( \frac{3y^{(r)} + 2}{3y^{(r)} + 2} \right)^{-1} \right\} \leq 1, \\ \infty & \text{when } \frac{\bar{\sigma}_{eq}^2}{\bar{\mu}} \sup_{y^{(r)} \geq 0} \left\{ F \left( \frac{\mu^{(r)}}{\bar{\mu}} \right) \left( \frac{3y^{(r)} + 2}{3y^{(r)} + 2} \right)^{-1} \right\} > 1. \end{cases}
\]
(13)
Comparing the above equation with (7) we conclude that the composite material obeys a von-Mises-like yield criterion with a flow stress \( \bar{\sigma}_0 \) defined by
\[
\bar{\sigma}_0 = \inf_{y^{(r)} \geq 0} \left[ \sum_{r=1}^{N} c^{(r)}(3y^{(r)} + 2)^{-1} \left( \frac{3y^{(r)} + 2}{3y^{(r)} + 2} \right)^{-1} \left( \frac{\sigma^{(r)}_{0}}{c^{(r)}_{0}} \right)^n \right].
\]
(14)
This is the end of the third step.
The average deformation rate $\bar{D}^{(r)}$ over each phase is determined from the macroscopic deformation rate $\bar{D}$ in terms of a “strain concentration tensor” $A^{(r)}$:

$$\bar{D}^{(r)} = A^{(r)} \cdot \bar{D},$$

(15)

where in the present case

$$A^{(r)} = \frac{y^{(r)}}{3y^{(r)} + 2} \left( \sum_{s=1}^{N} c^{(s)} y^{(s)} \right)^{-1} \mathcal{K} \quad \text{for} \quad r = 2, 3, 4, \quad A^{(1)} = \frac{1}{c^{(1)}} \left( I - \sum_{r=2}^{4} c^{(r)} A^{(r)} \right),$$

(16)

$\mathcal{K} = I - J$, $J = \frac{1}{3} \delta \delta$, $\delta$ is the second-order unit tensor, and $I$ the symmetric forth-order unit tensor with Cartesian components $I_{ijkl} = \frac{1}{2} \left( \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} \right)$. The evolution of the volume fractions of the phases due to martensitic transformation is calculated as described by Triantafillou et al. [4].

The above formulation is rigorous when all phases are perfectly plastic. In our applications, the analysis is carried out incrementally and the yield stresses of the phases are updated at each increment.

Note that the calculation of the effective yield stress $\bar{\sigma}_0$ in (14) involves a constrained minimization problem. This is solved by using the methodology developed by Kaufman et al. [5] and the CONMAX software (http://www.netlib.org/opt/conmax.f).

A methodology for the numerical integration of the constitutive model is developed and implemented in the ABAQUS general purpose finite element program. This code provides a general interface so that a particular constitutive model can be introduced as a “user subroutine” (UMAT).

3. Applications
3.1 The example of a two-phase composite

In order to check the proposed model, we consider a simple two-phase system with $c^{(1)} = 0.6$, $c^{(2)} = 0.4$, and $\sigma_0^{(2)} / \sigma_0^{(1)} = 1.3$, phase-1 being the matrix material. The prediction of the model for uniaxial tension loading is compared with detailed unit-cell finite element calculations in Fig. 1. The unit cell is cylindrical with a spherical inclusion at its center.
3.2 Forming limit diagrams

We consider a sheet made of TRIP steel that is deformed uniformly on its plane in such a way that the in-plane principal strain increments increase in proportion. We study the possibility of the formation of a neck in the form of a narrow straight band and construct the corresponding “forming limit diagram”. The details of the formulation are given in Papatriantafillou et al. [4].

The results presented in the following are from the rate-dependent version of the model with a large strain-rate exponent, so that the material is very close to rate-independent. We follow the approach of Marciniak and Kuzynski [6], known as the “M–K” model, in which the sheet is assumed to contain a small initial inhomogeneity and necking results from a gradual localization of the strains at the inhomogeneity. The inhomogeneity is in the form of straight narrow band (neck) of reduced thickness $H^b < H$. Both inside and outside the band a state of uniform plane stress is assumed, and the analysis consists in determining the uniform state of deformation inside the band that is consistent kinematically and statically with the prescribed uniform state outside the band. Given the initial sheet thickness inside and outside the band and the imposed uniform deformation history outside the band, the equations of equilibrium are solved incrementally to obtain the deformation history inside the band. Localization is said to occur when the ratio of some scalar measure of the amount of incremental straining inside the band to the corresponding value outside the band becomes unbounded. The material data and the details of the calculations are described in Papatriantafillou et al. [4].

Figure 2 shows “forming limit curves” obtained for imposed proportional straining for two different values of the initial thickness imperfection, namely $H^b / H = 0.999$ and 0.99. In particular, the curves in Fig. 2 show the values of the strains $\varepsilon_1 = \varepsilon_1^{cr}$ and $\varepsilon_1^{cr} = \rho \varepsilon_2^{cr}$ at which necking takes place for different values of $\rho$. The two solid curves correspond to the TRIP steel, whereas the dashed curves are for the non-transforming steel. The TRIP phenomenon increases the necking localization strains. In particular, for an initial thickness imperfection of $H^b / H = 0.99$ and $\rho = 0$ (plane strain), the critical strain $\varepsilon_1^{cr}$ increases from 0.2145 for the non-transforming steel to 0.2541 for the TRIP steel; the corresponding values of $\varepsilon_1^{cr}$ for $H^b / H = 0.999$ and $\rho = 0$ are 0.3179 for the non-transforming steel and 0.3567 for the TRIP steel.

Fig. 2 Forming limit curves for two different values of initial thickness inhomogeneities $H^b / H = 0.999$ and 0.99. The solid lines correspond to the TRIP steel and the dashed lines are for a non-transforming steel. The dark triangles are experimental data.
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